Book Reviews

J. Michael Dunn and Gary M. Hardegree, **Algebraic Methods in Philosophical Logic**, Oxford Logic Guides, no.41, Clarendon Press, Oxford University Press, Oxford, New York, etc., 2001, pp xv + 470.

This is certainly a very interesting book including lots of material written over a long period. The authors goal was to "represent a reasonable portion of the intersection of algebraic logic and philosophical logic". The objective was achieved and so the book may be useful for a wide audience that includes, as the authors say in the preface, mathematicians, philosophers, information scientists and maybe linguists.

This book has already been reviewed by J. Czelakowski (*The Bulletin of Symbolic Logic*, Volume 9, Number 2, June 2003) so I will not repeat, for instance, some "complementary remarks" that the mentioned reviewer states in p. 233.

The book has 13 chapters and is mostly the work of the first author. The second author has only written chapter 9, chapters 4–7 being joint work.

Only propositional logic is considered, no attention paid to first-order logic as "perhaps the authors know too little". Besides that, very little space is devoted to relevance and quantum logic as "the authors know too much".

The book begins with an only nine pages long and quite readable introduction although the authors warn that the reader who is completely new to algebraic logic may find it difficult to understand. Boole's approach and the Frege approach to logic are compared. Afterwards, the authors present a brief analysis of the algebras of the syntax and the semantics of propositional logics arriving to the completeness concept. Finally we are led to representation theory and its connection with completeness.

Then comes a chapter on Universal Algebra (45 pp.) including material such as Birkhoff's Varieties Theorem.

A chapter on order, lattices and Boolean Algebras follows (60 pp.). Order algebras and tonoids are also considered towards obtaining the right framework to analyze logics. In a tonoid every operation has a tonic type ($+$ or $-$) according to which the operation is isotonic or antitonic in every one of its arguments.

Chapter 4 is short and deals with Syntax. The authors discuss three different but equivalent ways of defining a language, but decide in what follows to see language as a universally free algebra.

In chapter 5 on Semantics the relation between sentences and propositions is studied. The authors use the concepts of logical matrix and logical atlas as fundamental to their general algebraic scheme. A logical atlas is a generalization of a logical matrix. A logical atlas is defined to be an algebra together with a non-empty family of non-empty proper subsets of the carrier set of the algebra (the authors will devote the whole of chapter 7 to the study of matrices and atlases). Then it is introduced the very important philosophical and mathematical distinction between valuations and interpretations: valuations are functions giving a truth value (which can be seen philosophically as Fregean references) and interpretations are homomorphisms (reflecting compositionality) assigning what can philosophically be seen as a Fregean sense. The authors then distinguish between evaluationally constrained and locally or globally evaluated interpreted languages and also consider other similar distinctions. Many notions of logic are expressed in the context of an evaluationally constrained language. Finally, Section 5.14 entitled "The Three-Fold way" may be of special interest to philosophers. There the authors apply the atlas concept to usual suggestions in logical texts as regards the nature of logical validity: either a matter of logical form or a matter of logical necessity or a combination of both.

In the following chapter, entitled "Logic", the authors take the view that "consequence" is the central business of logic, a point of view that sometimes has been obscured, when the "laws" of logic have been emphasized instead, that is, the systems that the authors call unary assertional. They discuss four ways of presenting logical systems: the unary assertional, binary implicational, asymmetric consequence and symmetric consequence systems. In the case of the last concept the authors build on the work of Dana Scott, Shoesmith and Smiley. Emphasis is given on the property of absoluteness of the relation of symmetric consequence, in the sense that it has precisely one semantics. Section 10 on Structural (Formal) Consequence may be, again,

of special interest to philosophers, as the authors deal briefly on the question whether all consequence can be reduced to formal consequence.

Chapter 8 is devoted to representation theorems, emphasizing propositions as sets of information states. Chapters 9–11 deal respectively with classical propositional, modal and intuitionistc logic.

Chapter 12 deals again with tonoids and also with gaggles, distributoids and partial gaggles in order to consider "a class of algebras most generally appropriate for the study of both classical and non-classical logics".

In Chapter 13 the authors deal with the phenomenon of duality for Boolean algebras and distributive lattices, proving basically two theorems, considering a Stone space in the case of Boolean algebras and a Priestley space in the case of distributive lattices.

As regards mistakes, Czelakowski has mentioned a number of them. Here we may add that not only Theorem 3.6.13 (as stated by Czelakowski) but also Theorem 3.6.14 (p. 69) seems useless because it is trivial using definition 3.6.2 in p. 68. There are also small mistakes, which the reader probably easily recognizes as such, for example, in the Introduction, p. 4, in the middle of the second paragraph, $[\varphi] \vee [\varphi]$ and $[\varphi]$ are equivalence classes, so "[" and "]" should be omitted, as the authors are referring to sentences.

As regards the style, it is very helpful for understanding. The authors provide clear definitions and usually give concrete examples, even non-examples and many exercises in order to familiarize the reader with abstract notions. Occasionally they repeat definitions in order that the reader is not always forced to look for the place where they were originally stated. The authors take care to provide many distinctions of the concepts involved, what helps understanding. Also interesting historical references are given.

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